G53RDB: Theory of Relational Databases

Lecture 14

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Plan of the lecture

- More Datalog:
  - Safe queries
  - Datalog and relational algebra
- Recursive Datalog rules
- Semantics of recursive Datalog rules
- Problems with negation
- Stratified Datalog

Datalog syntax: rules

- A Datalog ***rule*** is an expression of the form

$$R_1 \leftarrow R_2 \text{ AND } \ldots \text{ AND } R_n$$

where $n \geq 1$, R_1 is a relational atom, and $R_2,\ldots, R_n$ are relational or arithmetic atoms, possibly preceded by NOT.

- R_1 is called the ***head*** of the rule and $R_2,\ldots, R_n$ the ***body*** of the rule.
- $R_2,\ldots, R_n$ are called ***subgoals***.

Example

- Suppose we have a relation Person over schema (Name, Age, Address, Telephone). Then the following Datalog rule will define a relation which contains names of people aged over 18:

$$\text{Adult}(x) \leftarrow \text{Person}(x,y,z,u) \text{ AND } y \geq 18$$

Datalog query

- A ***Datalog query*** is a finite set of Datalog rules
- If there is only one relation which appears as a head of a rule in the query, the tuples in that relation are taken as the answer to the query.
- For example,

$$\text{Parent}(x,y) \leftarrow \text{Mother}(x,y)$$
$$\text{Parent}(x,y) \leftarrow \text{Father}(x,y)$$

defines Parent relation (using relations Father and Mother)

- If there is more than one relation appearing as a head, one of them is the main predicate to be defined and others are auxiliary.

Meaning of Datalog rules

- First approximation (non-recursive queries):
  - take the values of variables which make the body of the rule true (make each subgoal true; NOT R is true if R is false)
  - see what values the variables of the head take;
  - add the resulting tuple to the predicate in the head of the rule.

Example with negation

- Suppose we have a relation Person over schema (Name, Age, Address, Telephone).

  Child(x) $\leftarrow$ Person(x,y,z,u) AND NOT($y \geq 18$)
- We take all <name, age, addr, tel> in Person for which it is also true that NOT(age ≥ 18), and add <name> to Child.
- NOT(age ≥ 18) is true if age ≥ 18 is false, so we add all tuples where age < 18.

Safe queries

- We want the result of a query to be a finite relation.
- To ensure this, the following ***safety condition*** is required:

every variable that appears anywhere in the rule must appear in some non-negated relational subgoal.

- The reason for this is that infinitely many values may satisfy an arithmetical subgoal (e.g. $x > 0$) and infinitely many values are NOT in some finite table of a relation R.

Questions

- Which of the following rules have safety violations:
  - P(x,y) $\leftarrow$ Q(x,y) AND NOT R(x,y)
  - P(x,y) $\leftarrow$ NOT Q(x,y) AND y = 10
  - P(x,y) $\leftarrow$ Q(x,z) AND NOT R(w,x,z) AND x < y
  - P(x,y) $\leftarrow$ Q(x,z) AND R(z,y) AND NOT Q(x,y)

Questions

- Which tuples are in P?

  P(x,y) $\leftarrow$ Q(x,z) AND R(z,y) AND NOT Q(x,y)

given that:

Q contains tuples <a,b>, <a,c>

R contains tuples <b,c>, <c,a>

Datalog and relational algebra

- Every relation definable in relational algebra is definable in Datalog.
- Again we assume that we have a relational name (predicate symbol) R for every basic relation **R**.
- Then for every operation of relational algebra, we show how to write a corresponding Datalog query.

Union

- Union of **R** and **S**:

$U(x_1,\ldots,x_n) \leftarrow R(x_1,\ldots,x_n)$

$U(x_1,\ldots,x_n) \leftarrow S(x_1,\ldots,x_n)$

Difference

- Difference of **R** and **S**:

$D(x_1,\ldots,x_n) \leftarrow R(x_1,\ldots,x_n)$ AND NOT $S(x_1,\ldots,x_n)$

Product

- Product of **R** and **S**:

$P(x_1,\ldots,x_n,y_1,\ldots,y_k) \leftarrow R(x_1,\ldots,x_n)$ AND $S(y_1,\ldots,y_k)$

Projection

- Suppose we want to project **R** on attributes $x_1,\ldots,x_n$.

$P(x_1,\ldots,x_n) \leftarrow R(x_1,\ldots,x_n,y_1,\ldots,y_k)$

or

$P(x_1,\ldots,x_n) \leftarrow R(x_1,\ldots,x_n,\_,\ldots,\_)$

Selection

- Simple case: all conditions in the selection are connected by AND, for example $\sigma_{\textbf{Age > 18 AND Address = "London"}}$ (Person)

Answer(x,y,z,u) ← Person(x,y,z,u) AND y > 18 AND z = "London"

- If conditions are connected with OR, need more than one rule. For example, $\sigma_{\textbf{Age > 18 OR Address = "London"}}$ (Person)

Answer(x,y,z,u) ← Person(x,y,z,u) AND y > 18
Answer(x,y,z,u) ← Person(x,y,z,u) AND z = "London"

Compound queries

- To translate an arbitrary algebraic expression, create a new predicate for every node in the query tree.
- For example, to do $\sigma_{\text{Name1 = Name2}}$ $(R \times P)$:
  - Define predicate $S = R \times P$
  - Define $\sigma_{\text{Name1 = Name2}}$ (S)

Recursion: motivating example

- Consider a database for London underground.
- It describes lines, stations, station closures etc. (there may be stations closed on weekends, or because of technical problems or strikes).
- Typical queries include:
  - is it possible to go from King's Cross to Embankment?
  - which lines can be reached form King's Cross?

Motivating example

- We can either compute and store this information for every station (recompute it every day because of station closures)
- Or, we can store the basic data (Links relation below) and compute answers to queries as they are asked.

Links

Line	Station	Next Station
Central	Marble Arch	Bond St
Jubilee	Bond St	Green Park
Victoria	Green Park	Victoria
Victoria	Victoria	Pimlico

Motivating example

- However, in a relational database, given a relation Links, we cannot express a query "Is Pimlico reachable from Marble Arch?".

Links

Line	Station	Next Station
Central	Marble Arch	Bond St
Jubilee	Bond St	Green Park
Victoria	Green Park	Victoria
Victoria	Victoria	Pimlico

Recursive queries

- Reachability in a graph is a typical recursive property.
- It cannot be expressed in relational calculus or relational algebra given an Edge relation for the graph.
- We can write a query which expresses "reachable in one step", "reachable in two steps", and so on, but not simply "reachable".
- Another example: given a Parent relation, write a query which finds ancestors of a given person.
- Again, in relational algebra or calculus we can find parents, grandparents and so on, but not all ancestors.

Example recursive program

Reachable (x,x) ←

Reachable (x,y) ← Links(u,z,y) AND Reachable (x,z)

- We use the database relation Links to define relation Reachable, which is not stored in the database.
- To compute the set of stations reachable from King's Cross, we add to this program

Answer(y) ← Reachable("King's Cross", y)

Extensional and intensional predicates

- To distinguish relations which are in the database and relations which are being defined by Datalog rules:
  - ***Extensional*** predicates: predicates whose relations are stored in a database
  - ***Intensional*** predicates: defined by Datalog rules
- EDB – extensional database – collection of extensional relations
- IDB – intensional database – collection of intensional relations

Three ways to give semantics of recursive Datalog programs

- Minimal relations (minimal models)
- Provability semantics
- Fixpoint semantics

For the time being, assume that we do not have negation on IDB predicates

Minimal relations

- Datalog programs are logical descriptions of new relations. The answer to the Datalog query is the smallest relation which satisfies all the stated properties.
- Each rule

$R_1(xs) \leftarrow R_2(xs)$ AND … AND $R_n(xs)$

- corresponds to a logical property

$\forall x_1 ... \forall x_m (R_2(xs) \& ... \& R_n(xs) \rightarrow R_1(xs))$

where $x_1,...,x_m$ are all the variables occurring in the rule and xs some subsequence of $x_1,...,x_m$.

Example

- A program

Ancestor(x,y) ← Parent(x,y)

Ancestor(x,y) ← Parent(x,z) AND Ancestor(z,y)

- corresponds to logical properties

P1 $\forall x \forall y$ (Parent(x,y) → Ancestor(x,y))

P2 $\forall x \forall y \forall z$(Parent(x,z) & Ancestor(z,y) → Ancestor(x,y))

Example

- Suppose Parent contains just two pairs:

Parent(Anne, Bob), Parent(Bob, Chris)

- Because of P1, Ancestor should contain the same pairs:

Ancestor(Anne, Bob), Ancestor(Bob, Chris)

- Because of P2, we also need to add Ancestor(Anne,Chris) to satisfy

$\forall x \forall y \forall z$(Parent(x,z) & Ancestor(z,y) → Ancestor(x,y))

Parent(Anne,Bob) & Ancestor(Bob,Chris) → Ancestor(Anne,Chris))

Programs as proofs

- Proof-theoretic way of looking at Datalog programs:
- for which tuples can we logically prove that they are in Ancestor relation (using Parent relation and the program rules).
- Happens to be the same tuples as in the minimal Ancestor relation.

Fixpoint semantics of programs

- Start assuming that all IDB predicates are empty.
- Construct larger and larger IDB relations by:
  – Fire rules to add a tuples to IDB relations
  – Use tuples added to IDB relations in the previous round to add a new tuples to IDB relations
- Continue firing rules until no new tuples are added (reached a ***fixpoint***). If rules are safe, there will be finitely many tuples which satisfy the body of the rule, so fixpoint will be reached after finitely many rounds.
- This happens to give the same answer as “what is the minimal relation satisfying the properties” and “for which tuples can we prove that they are in Ancestor relation”.

Example: fixpoint construction

Ancestor(x,y) ← Parent(x,y)

Ancestor(x,y) ← Parent(x,z), Ancestor(z,y)

- Start: Ancestor = {}, Parent={<a,b>,<b,c>,<c,d>}
- 1st round: Ancestor = {<a,b>,<b,c>,<c,d>}
- 2nd round: Ancestor = {<a,b>,<b,c>,<c,d>, <a,c>, <b,d>}

(Ancestor(a,c) ← Parent(a,b), Ancestor(b,c) gives <a,c>

Ancestor(b,d) ← Parent(b,c), Ancestor(c,d) gives <b,d>)

- 3rd round: Ancestor = {<a,b>,<b,c>,<c,d>, <a,c>,<b,d>,

Ancestor(a,d) ← Parent(a,b), Ancestor(b,d) <a,d>}

- 4th round: no new tuples in Ancestor.

Negation

- Problem with negation: may not be a unique minimal solution; no clear semantics.
- Example: EDB = {R} and IDB = {P,Q}

P(x) ← R(x) AND NOT Q(x)

Q(x) ← R(x) AND NOT P(x)

Suppose R = {<a>}. Then either

  – P = {<a>} and Q empty, or
  – Q = {<a>} and P empty.

No unique solution. Can’t say if P(a) holds or Q(a) holds.

Stratified Datalog with negation

- The idea is to break cycles as in the example before, when to evaluate IDB predicate P we need to know what is the negation of IDB predicate Q, and vice versa (P is defined using NOT Q and Q is defined using NOT P).
- Solution: outlaw cycles in dependencies on negative IDB predicates.

What does “depend” mean

- If R is the head of a rule where P is in the body, R depends on P
- If R is the head of a rule where P is in the body, and P depends on S, then R depends on S (transitive relation).
- We draw a dependency graph for IDB predicates.

What does “depend” mean

- (Only IDB predicates are shown, E assumed to be an EDB predicate). R depends on P, S, Q, T and V; P depends on S; Q depends on V and T; V depends on T and Q, and T depends on Q and V

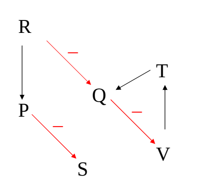
R(x) ← P(x) AND NOT Q(x)

Q(x) ← NOT V(x) AND E(x)

P(x) ← NOT S(x) AND E(x)

V(x) ← T(x)

T(x) ← Q(x)



What does “depend” mean

- Negative arcs (with – sign) correspond to negative occurrences of predicates in the body of the rule
- Recursion is ***stratified*** if there is no cycle involving negative arcs. (The program below is not stratified)

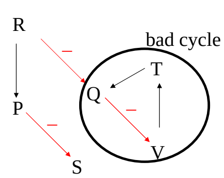
R(x) ← P(x) AND NOT Q(x)

Q(x) ← NOT V(x) AND E(x)

P(x) ← NOT S(x) AND E(x)

V(x) ← T(x)

T(x) ← Q(x)



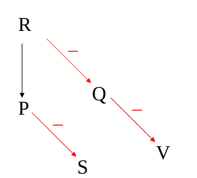
Strata

- In a stratified program, IDB predicates are divided into ***strata.***
- Stratum of a predicate is the maximal number of negative arcs on a dependency path starting at that predicate.

R(x) ← P(x) AND NOT Q(x)

Q(x) ← NOT V(x) AND E(x)

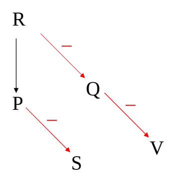
P(x) ← NOT S(x) AND E(x)



Example

- The program below is stratified.
- Stratum 0 = {S, V}
- Stratum 1 = {P, Q}
- Stratum 2 = {R}

R(x) ← P(x) AND NOT Q(x)
Q(x) ← NOT V(x) AND E(x)
P(x) ← NOT S(x) AND E(x)



In other words

- stratum 0: do not depend on any negated IDB predicates
- stratum 1: depend on negated IDB predicates from stratum 0;
- stratum 2: depend on negated IDB predicates from stratum 1,
- …
- stratum n: depend on IDB predicates from stratum n-1.

Evaluating stratified Datalog programs

- Stratified Datalog programs have the following operational semantics:
  - First compute all IDB predicates in stratum 0 (using the usual fixpoint strategy)
  - …
  - Using IDB predicates from stratum n, compute IDB predicates from stratum n+1.
- This produces unique minimal solutions for all IDB predicates.

Informal coursework

- Is the following program stratified (EDB = {S}):
  Q(x) ← NOT P(x) AND R(x)
  P(x) ← NOT R(x) AND S(x)
  R(x) ← S(x)
- Is the following program stratified (EDB = {S}):
  R(x) ← Q(x)
  Q(x) ← R(x)
  R(x) ← S(x) AND NOT Q(x)
- For the stratified program, compute P, Q and R given that S contains {<a>,<b>}.

Informal coursework

- A database of fictitious company contains three relations:
  - GOODS over schema {Producer, ProductCode, Description}
  - DELIVERY over schema {Producer, ProductCode, Branch#, Stock#}
  - STOCK over schema {Branch#, Stock#, Size, Colour, SellPrice, CostPrice, DateIn, DateOut}.

Define in Datalog

- Query 1: find all producers who supply goods.
- Query 2: find all producers who have delivered goods to any branch of the company.
- Query 3: find SellPrice and CostPrice of all goods delivered to branch L1 still in stock (here, L1 is a value in the attribute domain of Branch#, and products in stock have value InStock for the DateOut attribute).
- Query 4: find Producer, ProductCode, Description for all goods sold at the same day they arrived at any branch.
- Query 5: find Branch#, Size, Colour, SellPrice for all dresses which have not yet been sold (dress is a value in the attribute domain of Description).

Reading

- Ullman, Widom, chapter 10
- Abiteboul, Hull, Vianu chapter 12.